**Assignment-1**

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.

b) Miami is the capital of Florida.

c) 2+3 = 5.

d) 5+7 = 10.

e) x +2 = 11.

f) Answer this question.

2. What is the negation of each of these propositions?

a) Jennifer and Teja are friends.

b) There are 13 items in a baker’s dozen.

c) Abby sent more than 100 text messages every day.

d) 121 is a perfect square.

3. Let p and q be the propositions p : I bought a lottery ticket this week. q : I won the million dollar jackpot.

Express each of these propositions as an English sentence.

a) ¬p

b) p ∨q

c) p → q

d) p ∧q

e) p ↔ q

f) ¬p →¬ q

g) ¬p∧¬q

h) ¬p∨(p∧ q)

4. Let p and q be the propositions p :It is below freezing. q :It is snowing. Write these propositions using p and q and logical connectives (including negations).

a) It is below freezing and snowing.

b) It is below freezing but not snowing.

c) It is not below freezing and it is not snowing.

d) It is either snowing or below freezing (or both).

e) If it is below freezing, it is also snowing.

f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.

g) That it is below freezing is necessary and sufﬁcient for it to be snowing

5. Construct a truth table for each of these compound propositions.

a) p →¬ q

b) ¬p ↔ q

c) (p → q)∨(¬p → q)

d) (p → q)∧(¬p → q)

e) (p ↔ q)∨(¬p ↔ q)

f) (¬p ↔¬q)↔ (p ↔ q)

6. Show that¬ (¬p) and p are logically equivalent.

7. Use a truth table to verify the ﬁrst De Morgan law

¬ (p∧ q) ≡ ¬p∨¬q.

8. Determine whether (¬p (p → q))→¬ q is a tautology.

9. Determine whether (¬q ∧ (p → q))→¬p is a tautology.

10. Let Q(x) be the statement “x +1 > 2x.” If the domain consists of all integers, what are these truth values?

a) Q (0)

b) Q (−1)

c) Q (1)

d) ∃x Q(x)

e) ∀x Q(x)

11. Express each of these statements using quantiﬁers.

a) All dogs have ﬂeas.

b) There is a horse that can add.

c) Every koala can climb.

d) No monkey can speak French.

e) There exists a pig that can swim and catch ﬁsh.